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COLD BLACK HOLES IN SCALAR-TENSOR THEORIES

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Abstract

We study the possible existence of black holes in scalar-tensor theories of gravity in four dimensions. Their existence is verified for anomalous versions of these theories, with a negative kinetic term in the Lagrangian. The Hawking temperature $T_{\rm H}$ of these holes is zero, while the horizon area is (in most cases) infinite. It is shown that an infinite value of $T_{\rm H}$ can occur only at a curvature singularity rather than a horizon. As a special case, the Brans-Dicke theory is studied in more detail, and two kinds of infinite-area black holes are revealed, with finite and infinite proper time needed for an infalling particle to reach the horizon.

1. Introduction

This study was to a certain extent stimulated by a controversy in the recent literature: the paper by Campanelli and Lousto [1] asserts that in the well-known family of static, spherically symmetric vacuum solutions of the Brans-Dicke theory there exists a subfamily which possesses all properties of black hole solutions, but (i) these solutions exist only for negative values of the coupling constant ω and (ii) the horizons have an infinite area. These authors argue that large negative ω are compatible with modern observations and that such black holes may be of astrophysical relevance. On the other hand, H. Kim and Y. Kim [2], agreeing that there are non-Schwarzschild black holes in the Brans-Dicke theory, claim that such black holes have unacceptable thermodynamical and geometric properties and are therefore physically irrelevant; meanwhile, they ascribe such solutions to positive values of ω.

The aim of this work is not only to make the situation clear, but a bit wider: to reveal possible black hole solutions among static, spherically symmetric solutions of the general (Bergmann-Wagoner) class of scalar-tensor theories of gravity, which may be described in terms of the function $\omega(\phi)$; the Brans-Dicke theory ($\omega = \text{const}$) will be used just as an example. One of the reasons for such an approach is that, by modern views, it is rather probable that this coupling parameter could have been sufficiently small and could appreciably affect the physical processes in the early Universe, but by now became large, making the theory very close to general relativity in observational predictions.

To identify the presence of a regular horizon, we use such criteria as the finiteness of the Kretschmann scalar and the Hawking temperature. They have been used in [1, 2] as well, but we try to perform a more complete analysis. In particular, we prove that an infinite Hawking temperature implies the divergence of the Kretschmann scalar.

The case of the Brans-Dicke theory is studied in more detail. We find that regular event horizons do exist, but those with an infinite area (we call them Type B horizons), for negative values of $\omega + 3/2$, thus confirming the conclusions of [1].

Moreover, we show that black holes can exist in other theories, where the coupling $\omega \neq \text{const}$. In all such cases, the Hawking temperature is also zero (cold black holes) and the horizon area must be infinite in most cases (except $k < 0$, see Sec. 3).

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2. The Kretschmann Scalar and the Hawking Temperature

We would like to begin with some general consideration of static, spherically symmetric space-times, with results to be used in the subsequent sections.

The general form of the metric of such space-times is

$$ds^2 = \mathrm{e}^{2\gamma} dt^2 - \mathrm{e}^{2\alpha} du^2 - \mathrm{e}^{2\beta} d\Omega^2 \,, \tag{1}$$

where γ, α and β are functions of u only and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$.

Event horizons, to be discussed below, must be regular, hence all curvature invariants must be finite there. The finiteness of the Kretschmann invariant $R^{\mu\nu\lambda\gamma}R_{\mu\nu\lambda\gamma}$ is known to be the most efficient criterion of this type. We will, however, prove another, sometimes more convenient criterion: an infinite Hawking temperature indicates that an assumed horizon exhibits a singularity (in fact, in this case the use of the term "horizon" is doubtful).

The Kretschamnn scalar for the metric (1) may be written as

$$K = 4K_1^2 + 8K_2^2 + 8K_3^2 + 4K_4^2 \,, \tag{2}$$

where

$$\begin{aligned}
K_1 &= R^{01}{}_{01} = -\,\mathrm{e}^{-\alpha-\gamma}\Big(\gamma'\,\mathrm{e}^{\gamma-\alpha}\Big)', \\
K_2 &= R^{02}{}_{02} = R^{03}{}_{03} = -\,\mathrm{e}^{-2\alpha}\beta'\gamma', \\
K_3 &= R^{12}{}_{12} = R^{13}{}_{13} = -\,\mathrm{e}^{-\alpha-\beta}\Big(\beta'\,\mathrm{e}^{\beta-\alpha}\Big)', \\
K_4 &= R^{23}{}_{23} = \mathrm{e}^{-2\beta} - \mathrm{e}^{-2\alpha}\beta'^2
\end{aligned} \tag{3}$$

where a prime denotes d/du. The structure of Eq. (2) indicates that an infinite value of any K_i implies the presence of a singularity at a given point of the space-time.

Let us now prove the following statement:

- If, at a certain surface $u = u^*$ of a static, spherically symmetric configuration, $g_{00} = 0$ (a candidate horizon) and the Hawking temperature $T_{\rm H}$ calculated for this surface, is infinite, this surface is a curvature singularity.

Indeed, using e.g. formulae from the book [5], one finds for static metrics written in the form (1) the following expression for the Hawking temperature of a surface $u = u^*$ where $\mathrm{e}^{\gamma} = 0$, assumed to be a horizon:

$$T_{\rm H} = \frac{\kappa}{2\pi}, \qquad \kappa = \lim_{u\to u^*} e^{\gamma-\alpha}|\gamma'| \tag{4}$$

where we have put the Boltzmann constant $k_{\rm B}$ and the Planck constant $\hbar$ equal to 1. (The same expression can be obtained using other methods, such as Euclidean continuation of the metric).

Assume now that the surface $u = u^*$ is a candidate horizon of the metric (1), so that $\mathrm{e}^{2\gamma} \to 0$ when $u \to u^*$. Assume, in addition, that $\kappa = \infty$, while both functions $\gamma(u)$ and $\mathrm{e}^{\gamma-\alpha}\gamma'$ are monotonic in some neighbourhood of u^*. Let us show that then the Kretschmann scalar $K \to \infty$ as $u \to u^*$.

It is sufficient to prove that $K_1 \to \infty$.

Let us use the fact that K_1 (as well as other K_i) and the expression $\mathrm{e}^{\gamma-\alpha}|\gamma'|$ are unaffected by reparametrizations of the radial coordinate u. With this invariance, any coordinate conditon for u may be chosen without loss of generality. Let us choose the following one:

$$\gamma + \alpha = 0\,.$$

Then

$$K_1 = -\frac{1}{2}[2\gamma'\,\mathrm{e}^{2\gamma}]' = -\frac{1}{2}[\,\mathrm{e}^{2\gamma}]''.$$

By our assumptions, with (2.) we have $\mathrm{e}^{2\gamma} \to 0$ and $(\mathrm{e}^{2\gamma})' \to \infty$ as $u \to u^*$.

Let us denote $g(u) = \mathrm{e}^{2\gamma}$, $1/g'(u) = G(g)$. Then $G(g) \to 0$ as $g \to 0$. On the other hand, one can write:

$$\frac{dg}{du} = \frac{1}{G(g)} \quad \Rightarrow \quad u = \int G(g)dg.$$

This integral is evidently finite, hence u^* is finite in the coordinates (2.). Thus, for a finite value of u, we have $g' = dg/du \to \infty$, therefore

$$g'' \to \infty \qquad \Rightarrow \qquad |K_1| \to \infty,$$

which proves the statement.

3. Criteria for black hole selection

Black hole (BH) solutions with the metric (1) are conventionally selected by the following criteria: at some surface $u = u^*$ (horizon)

C1. $\mathrm{e}^{\gamma} \to 0$ (the timelike Killing vector becomes null);

C2. e^{β} is finite (finite horizon area);

C3. the integral $t^* = \int \mathrm{e}^{\alpha-\gamma} \to \infty$ as $u \to u^*$ (invisibility of the horizon for an observer at rest).

The evident requirement that a horizon must be regular (otherwise we deal with a singularity rather than a horizon) creates two more criteria:

C4. The Hawking temperature T_{H} is finite;

C5. The Kretschmann scalar K is finite at $u = u^*$.

As we have seen in Sec. 2, the condition C4 is necessary, but not sufficient for regularity, and this will be illustrated by the examples treated below. As for C5, the scalar K, due to its structure, is the most reliable probe for space-time regularity.

The condition C2 is apparently less essential than the others. In principle, C2 can be cancelled, leading to a generalized notion of a BH, that with a horizon having an infinite area, as described in [1]. We will call the BHs satisfying all the criteria C1–C5 type A black holes, and those with an infinite horizon — Type B black holes.

We shall see that in the general scalar-tensor theory (STT), and, in particular, in the Brans-Dicke theory all non-Schwarzschild black holes are type B, while the configurations satisfying C1–C3 turn out to be singular.

4. Possible Black Holes in the General Scalar-Tensor Theory

A general Lagrangian describing the interaction between gravity and a scalar field in four dimensions can be written as

$$L = \sqrt{-g}\Big(f(\phi)R + \frac{\omega(\phi)}{\phi}\phi_{;\rho}\phi^{;\rho}\Big), \tag{5}$$

where $f(\phi)$ and $\omega(\phi)$ are, in principle, arbitrary functions of the scalar field ϕ (the so-called Bergmann-Wagoner class of STT). Reparametrization of ϕ makes it possible to deal with one function, for instance, the most conventional formulation uses $f = \phi$. In what follows it is used here as well.

Performing the conformal mapping

$$g_{\mu\nu} = \phi^{-1}\bar{g}_{\mu\nu} \tag{6}$$

and omitting a total divergence, we obtain the Lagrangian

$$\bar{L} = \sqrt{-\bar{g}}\Big(\bar{R} + F(\phi)\phi_{;\rho}\phi^{;\rho}\Big) \tag{7}$$

where

$$F(\phi) = \frac{\omega + 3/2}{\phi^2} \tag{8}$$

The general static, spherically symmetric scalar-vacuum solution for the theory (5) is given by [3, 4]

$$ds^2 = \frac{1}{f}\bigg\{ \mathrm{e}^{-2bu}dt^2 - \frac{\mathrm{e}^{2bu}}{s^2(k,u)}\bigg[\frac{du^2}{s^2(k,u)} + d\Omega^2\bigg]\bigg\}, \tag{9}$$

$$F(\phi)\bigg(\frac{d\phi}{du}\bigg)^2 = S = \text{const}, \tag{10}$$

where the last expression is an integral of the scalar and gravitational field equations. The function $s(k,u)$ is defined as follows:

$$s(k,u) = \begin{cases} k^{-1}\sinh ku, & k > 0 \\ u, & k = 0 \\ k^{-1}\sin ku, & k < 0. \end{cases} \tag{11}$$

The constants b, k and S are connected by the relation

$$2k^2 \,\mathrm{sign}\, k = 2b^2 + S \tag{12}$$

which also follows from the gravitational equations. The constant S plays the role of a scalar charge.

For $k > 0$, after the coordinate transformation

$$\mathrm{e}^{-2ku} = 1 - \frac{2k}{r} \equiv P(r) \tag{13}$$

the metric can be rewritten as

$$ds^2 = \frac{1}{\phi}\Big[P^a dt^2 - P^{-a}dr^2 - P^{1-a}r^2 d\Omega^2\Big], \tag{14}$$

with the constants obeying the relation

$$S = 2k^2(1-a^2), \qquad a = b/k. \tag{15}$$

Let us analyze the possible existence of BHs in the general STT, i.e. with variable $\omega = \omega(\phi)$. As can be seen from (10), $S < 0$ corresponds to anomalous theories ($\omega + 3/2 < 0$), having a negative kinetic term in the Lagrangian (7), while for $S = 0$ we have $\phi = \text{const}$, i.e. general relativity.

From the viewpoint of Criteria C1–C3, there are four opportunities for BH existence [4]:

1. $k > 0$, $S < 0$, $u^* \to \infty$;

2. $k > 0$, $S > 0$, $u \to \infty$ is a regular sphere and a horizon may be found beyond it by proper continuation (example: a BH with a conformal scalar field);

3. $k = 0$, $S < 0$, $u^* \to \infty$;

4. $k < 0$, $S < 0$, $u^* = \pi/|k|$.

Let us consider each case separately, except for the second one, since it is hard to handle in a general form due to the continuation. We will try first to apply the requirements C1–C3. One can notice that in all cases to be considered the theory is anomalous[5].

1. $k > 0$. In this case we can use the metric (14) with (15). As $P \to 0$, due to C2 the scalar field behaves like
$$\phi \sim P^{1-a} \tag{16}$$
while from C3 it follows $a > 1$. The Hawking temperature is calculated from (4) and the asymptotic form of $\omega(\phi)$ may be found from (10). The result is
$$T_{\rm H} \sim \lim_{P\to 0} P^{a-1} = 0, \qquad \omega + \frac{3}{2} \to -\frac{1}{2}\frac{a+1}{a-1}. \tag{17}$$
However, the term K_1 of the Kretschmann scalar behaves as P^{-1} and tends to infinity as $P \to 0$.

2. $k = 0$. Using the metric (9) with $k = 0$, the conditions C1–C3 lead to $u^* = \infty$, and as $u \to \infty$,
$$\phi \sim \frac{1}{u^2}\,{\rm e}^{2bu}, \qquad b > 0, \tag{18}$$
For $T_{\rm H}$, ω and K_1 we obtain:
$$T_{\rm H} \sim \lim_{u\to\infty} u^2\,{\rm e}^{-2bu} = 0, \qquad \omega \to -2; \qquad K_1 \sim 4b^2u^2 \to \infty. \tag{19}$$

3. $k < 0$. A possible horizon is at $u^* = \pi/|k|$, where $\phi \sim 1/\Delta u^2$, $\Delta u \equiv |u - u^*|$. A calculation gives:
$$\begin{array}{l} |\,{\rm e}^{\gamma-\alpha}\gamma'| \sim \Delta u \to 0 \quad \Rightarrow \quad T_{\rm H} = 0; \\ \dfrac{\phi_u}{\phi} \sim \dfrac{1}{\Delta u} \to \infty \quad \Rightarrow \quad \omega + \dfrac{3}{2} \to -0. \end{array} \tag{20}$$
The scalar K is in this case finite; the behaviour of g_{00} and g_{11} near the horizon is similar to that in the extreme Reissner-Nordström solution.

We notice that in all these cases the Hawking temperature calculated for the assumed horizons is zero. However, $T_{\rm H} < \infty$ is only necessary but not sufficient for regularity. In the above cases of the general STT, the Kretschmann scalar turns out to be infinite and reveals a singularity in cases 1 and 3, while $K < \infty$ only in case 4 ($k < 0$). Thus for $k \geq 0$ the surfaces of finite area, satisfying the conventional criteria of an event horizon, turn out to be singular, and only Type B black holes can exist. This will be demonstrated explicitly for the Brans-Dicke theory.

5. Brans-Dicke Black Holes

Let us now analyze the four-dimensional spherically symmetric solution in Brans-Dicke theory ($\omega =$ const). In this case we can write explicitly $\phi = \phi_0\,{\rm e}^{su}$, where ϕ_0 and s are constants and $s^2 = S/(\omega + 3/2)$. The solution (9), (10) for different k then coincides (up to reparametrization) with different classes of the Brans solution [7] to the vacuum Brans-Dicke equations.

Let us again consider the cases $k > 0$, $k = 0$ and $k < 0$ separately.

[5] As shown in [4], the cases when the sphere $u = \infty$ is regular and admits an extension of the static coordinate chart, are very rare, and even when it is the case, such configurations turn out to be unstable due to blowing-up of the effective gravitational coupling [6].

5.1. $k > 0$

Using, as before, the transition (13) and denoting $\xi = s/2k$, we can write the metric in the form (14), namely,

$$ds^2 = P^{-\xi}\Big(P^a dt^2 - P^{-a}dr^2 - P^{1-a}r^2 d\Omega^2\Big), \tag{21}$$

and the constants are related by

$$(2\omega + 3)\xi^2 = 1 - a^2. \tag{22}$$

The limit $u \to \infty$ corresponds to $r \to 2k$, or $P \to 0$. The condition C1 ($g_{00} \to 0$ as $P \to 0$) implies $a - \xi > 0$. On the other hand, the condition C5 with $\xi \neq 0$ leads to $a + \xi \geq 2$. Indeed, we have

$$K_1 \sim (1 - a)(a - \xi)P^{a+\xi-2}. \tag{23}$$

With (22) this implies, in particular, $2\omega + 3 < 0$ (anomalous theory). Combining the two inequalities for a and ξ, we obtain the following allowed range for these constants:

$$a > 1, \qquad\qquad a > \xi \geq 2 - a. \tag{24}$$

A finite limit of g_{22} (Criterion C2) as $P \to 0$ is obtained if $a + \xi = 1$, and the only nonsingular case ($K < \infty$) under this condition is $\xi = 0$, $a = 1$ — the case when the Brans-Dicke theory reduces to general relativity and the solution is just the Schwarzschild metric. (In what follows we exclude it from consideration.) Moreover, the solution with $a + \xi = 1$ violates Criterion C3 when $\xi > 0$ since $t^* \sim \int P^{\xi-1}dr$, which converges in this case. Thus the non-Schwarzschild surface $P = 0$ is not only singular, but visible from outside for an observer at rest if $\xi > 0$, i.e., $\omega > -2$.

Let us now calculate the Hawking temperature for the solution (21). We have $\mathrm{e}^{\gamma-\alpha} \sim P^a$, $\gamma' \sim P^{-1}$ and consequently

$$T_{\rm H} \sim P^{a-1}\Big|_{P\to 0} = 0 \tag{25}$$

in the allowed range (24).

In the above case of finite limiting surface area ($a + \xi = 1$) one has $T_H = \infty$ if $a < 1$, $\xi > 0$ (this is just a visible naked singularity), but $T_H = 0$ if $a > 1$. But, as we remember, the Kretschmann scalar is infinite if $a + \xi < 2$. This is a clear illustration of the fact that a finite and even zero Hawking temperature does not guarantee a non-singular horizon.

In the allowed range (24) Criteria C1, C3–C5 hold, but the horizon radius is infinite and $T_{\rm H} = 0$ — a Type B black hole. Let us try, for better understanding of its properties, to perform a Kruskal-like extension of this solution beyond the horizon, i.e., to find a coordinate chart without an apparent metric singlularity $g_{00} = 0$ at $r = 2k$. To this end, let us first introduce the null coordinates v and w

$$v = t + x, \qquad\qquad w = t - x \tag{26}$$

where

$$x = \int dr/P^a, \tag{27}$$

so that $x \to \infty$ as $r \to \infty$ and $x \to -\infty$ as $r \to 2k$. The metric (21) takes the form

$$ds^2 = P^{a-\xi}dvdw - P^{1-a-\xi}r^2 d\Omega^2. \tag{28}$$

The integral (27) admits a closed expression only for integer a:

$$x = r + 2ka\ln(r - 2k) + \sum_{n=1}^{a-1}\frac{(-2k)^{n+1}}{n}\binom{a}{n+1}\frac{1}{(2k-r)^n}. \tag{29}$$

In the general case, the asymtotic behaviour of x as $r \to 2k$ is

$$x \sim (r - 2k)^{1-a}, \qquad\qquad a > 1. \tag{30}$$

Our next step must be a transition to some coordinates $V = V(v)$ and $W = W(w)$ eliminating the zero limit of g_{vw} in (28) as $x \to -\infty$, or, equivalently, as $v \to -\infty$ and/or $w \to \infty$.

Let us look for the new coordinates in the following asymptotic form:

$$v \sim V^p, \qquad\qquad w \sim W^p \tag{31}$$

with some constant p to be determined from the regularity requirement (it is the same for v and w from symmetry considerations). Assuming $v \to -\infty$ and finite w, we find the asymptotic form of the (U, V) part of the metric as

$$(r - 2k)^{1-\xi+(1-a)/p} dV\, dW, \tag{32}$$

which provides regularity at the horizon if

$$p = (1 - a)/(1 - \xi), \tag{33}$$

provided $\xi \neq 1$. The same expression for p is found if we consider the limit $w \to \infty$ keeping v finite.

With (33), V is asymptotically related to $r - 2k$ by

$$V \sim (r - 2k)^{1-\xi} + \mathrm{const.} \tag{34}$$

Hence for $\xi > 1$, $V \to \infty$ as $v \to -\infty$, while for $\xi < 1$ the coordinate V has a finite limit at the horizon. The same applies to the coordinate W. We see that the allowed range of ξ (see (24)) is divided into two subdomains by the line $\xi = 1$.

If $\xi = 1$, the assumption (31) does not work and the regularity is achieved using the transformation

$$V \sim \ln |v|, \qquad W \sim \ln |w|. \tag{35}$$

The coordinates V and W tend to infinity as $x \to -\infty$.

Thus for $\xi < 1$, we obtain a more or less common picture of a BH, where particles can arrive at the horizon in a finite proper time, and in principle they can cross it, entering the BH interior. Let us call such configurations Type B1 black holes.

When $\xi \geq 1$, the metric (32) behaves asymptotically as $dV\, dW$ with an infinite range of V and W, i.e., the assumed horizon is infinitely far and it would take an infinite time for any particle to reach it. Recalling that, in the same limit, $g_{22} \to \infty$, this configuration (to be called a Type B2 black hole) resembles a wormhole, although, in general, the correct flat-space asymptotic conditions are here not observed.

This interpretation is confirmed by a direct study of radial geodesics. The equation for a radial timelike trajectory in a static space-time with the metric (1) can be written, after first integration, as

$$\left(\frac{dr}{d\tau}\right)^2 = \mathrm{e}^{-2\alpha}(E^2\, \mathrm{e}^{-2\gamma} - 1), \tag{36}$$

where E is an integration constant (energy) and τ is the proper time. Close to a horizon ($\mathrm{e}^{\gamma} \to 0$) the second term can be neglected as compared with the first one. In our case $\mathrm{e}^{-\alpha-\gamma} \sim P^{2\xi}$, and leads to

$$d\tau \sim (r - 2k)^{-\xi}. \tag{37}$$

Hence a particle needs an infinite proper time to reach $r = 2k$ when $\xi \geq 1$ and a finite proper time when $\xi < 1$, in full agreement with the previous analysis of the metric behaviour.

One can notice that, under fixed a, large values of $|\omega|$, necessary to conform with present-day observations, correspond to small ξ, i.e. to Type B1 BHs with horizons available in finite proper times.

5.2. $k = 0$

In this case the metric (9) takes the form

$$ds^2 = \mathrm{e}^{-su}\left[\mathrm{e}^{-2bu}dt^2 - \frac{\mathrm{e}^{2bu}}{u^2}\left(\frac{du^2}{u^2} + d\Omega^2\right)\right], \qquad s^2(\omega + 3/2) = -2b^2. \tag{38}$$

There is no case of finite g_{22} as $u \to \infty$. One has

$$\gamma' \,\mathrm{e}^{\gamma-\alpha} = -u^2(b + s/2)\,\mathrm{e}^{-2bu}, \tag{39}$$

$$K_1 \sim b(b + s/2)u^4\,\mathrm{e}^{(s-2b)u}\,. \tag{40}$$

Thus $K_1 \to \infty$ if the radius $r = \sqrt{|g_{22}|} \to 0$ and $K_1 \to 0$ if $r \to \infty$ as $u \to \infty$. The case $b = 0$ is trivial, while $b + s/2 = 0$ corresponds to a "force-free" ($g_{00} = \mathrm{const}$) solution with a special value $\omega = -2$. The allowed range of the integration constants, where $g_{00} \to 0$ and $K_1 < \infty$, is

$$b > 0, \qquad\qquad 2b > s > -2b. \tag{41}$$

In this range the other $K_i < \infty$ as well. The Hawking temperature for these solutions is again zero and $|2\omega + 3| > 1$. These solutions, regular as $u \to \infty$, describe Type B black holes.

It is hard to obtain an explicit (even asymptotic) expression for a coordinate transformation needed for a Kruskal-like extension, since now $x = \int du \cdot \mathrm{e}^{2bu}/u^2$. We can, however, distinguish B1 and B2 BHs, using, as before, the geodesic equation (36). One has

$$d\tau \sim u^{-2}\,\mathrm{e}^{-su}du, \tag{42}$$

so that τ is finite for $s > 0$ and infinite for $s < 0$ ($s = 0$ is excluded since leads to general relativity). Thus the range (41) is again divided into two halves: for $s > 0$ we deal with a Type B1 BH, for $s < 0$ - with that of Type B2.

According to (38), large values of $|\omega|$ correspond to small s, which may be, however, of either sign, so that both Type B1 and B2 BHs can in principle be compatible with observations.

5.3. $k < 0$

Now the solution (9) is

$$ds^2 = \mathrm{e}^{-su}\left[\mathrm{e}^{-2bu}dt^2 - \frac{k^2\,\mathrm{e}^{2bu}}{\sin^2 ku}\left(\frac{k^2du^2}{\sin^2 ku} + d\Omega^2\right)\right], \qquad s^2(\omega + \frac{3}{2}) = -k^2 - 2b^2. \tag{43}$$

This is a wormhole solution [3], with two flat asymptotics at $u = 0$ and $u = u^* = \frac{\pi}{|k|}$. Due to the monotony of g_{00}, the masses at $u = 0$ and $u = u^*$ have opposite signs; there is neither a singularity, nor a horizon.

6. Concluding remarks

We can make the following general observations from the above analysis:

1. Black holes exist in anomalous scalar-tensor theories, i.e., when the kinetic term of the scalar field is negative, contrary to some statements put forward in [2].

2. For $k \geq 0$, no conventional (Type A) BHs can exist. BHs with an infinite area (Type B) do exist, as confirmed explicitly for a special case — the Brans-Dicke theory. It is shown that they in turn split into two classes, B1 and B2, with, respectively, finite and infinite proper time needed for an infalling particle to reach a horizon.

3. In the case $k < 0$ Type A black holes can exist, but only in theories with variable ω, and such explicit examples are yet to be found.

4. Type B2 BHs are obtained only in cases when the Brans-Dicke scalar field $\phi \to 0$ at the horizon, i.e., when the effective gravitational coupling tends to infinity.

Trying to perform an explicit Kruskal-like extension beyond the regular horizons (which makes sense for Type B1 BHs, available for particles), one meets, in the general case, negative quantities in fractional powers, and only in special cases when those powers are integer, the extension is performed using the well-known methods. The situation needs further study and we hope to return to it in future works.

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